$$H=\sum_k a_k^\dagger\left(\frac{k^2}{2m}-E_F\right)a_k+\frac{1}{2L}\sum_{k,k',q\neq 0}V(q)a_{k-q}^\dagger a_{k'+q}^\dagger a_{k'}a_k$$

$$\rho=\sum_k a_{k+q}^\dagger a_k$$ 粒子空穴激发

$$[\rho_{\sigma,q},\rho_{\sigma',-q'}]=-\delta_{\sigma,\sigma'}\delta_{q,q'}\frac{\sigma qL}{2\pi}$$

$$b_q=n_q\rho_{+,q},\quad b_q^\dagger=n_q\rho_{+,-q}\qquad n_q=\left(\frac{2\pi}{L|q|}\right)^{\frac{1}{2}}$$

$$b_{-q}=n_q\rho_{-,-q},\quad b_{-q}^\dagger=n_q\rho_{-,q}$$

$$H=v\sum_q|q|b_q^\dagger b_q$$

v速度电荷密度波

一维相互作用系统可以承载密度波激发，费米子部分改变系统的密度

$$N=\frac{L}{2\pi}2k_F\rightarrow n_0=\frac{k_F}{\pi}$$

此基础上可以定义密度涨落和相位，有下变换（正则变化）

$$b(x)=\left(\frac{k_F}{\pi}+\rho(x)\right)^{\frac{1}{2}}e^{i\varphi(x)}\qquad \left(\text{平移 } \rho\rightarrow\rho-\frac{k_F}{\pi}\right)$$

$$1=[b,b^\dagger]=[\rho^{\frac{1}{2}}e^{i\varphi},e^{-i\varphi}\rho^{\frac{1}{2}}]=\rho-e^{i\varphi}\rho e^{i\varphi}=\rho-e^{-i[\varphi,\,]}\rho$$

$$=\rho-\rho+i[\varphi,\rho]-\frac{1}{2}[\varphi,[\varphi,\rho]]+\cdots$$

$$\Rightarrow\quad [\rho,\phi]=i\qquad [\rho(x),\varphi(x')]=i\delta(x-x')$$

Jordan-Wigner 变换

$$a^\dagger(x)=e^{i\pi\int_{x'<x}dx'\,n(x')}b^\dagger(x)\qquad n(x)=b^\dagger(x)b(x)$$

$b^\dagger(x)$ — 玻色子产生算符

定义 $\theta(x) = \pi \int_{-\infty}^{x} dx' \rho(x')$　　ρ的正则对易是φ,且对应平移算符$e^{i\varphi(x)}$在x处产生一个单位的电荷

$$a^\dagger(x) = A \sum_\sigma e^{i\sigma\theta(x)} \quad e^{i\varphi(x)}$$

$\rho \to \rho + \frac{k_F}{\pi}$　　$a^\dagger(x) = A \sum_\sigma e^{i\sigma\pi^{-1}[\theta(x)+k_F x]} e^{i\varphi(x)}$

$\theta \to \theta + k_F x$　　$[\varphi(x), \theta(x')] = i\pi\Theta(x'-x)$

添加密度扰动　$\rho = \partial_x\theta/\pi$ 对应的能量 (短程局域,保证 1+1D 旋转不变性

$$S_0[\theta] = \frac{c}{2}\int dx d\tau \left((\partial_\tau\theta)^2 + (\partial_x\theta)^2 \right)$$

通过算　$\langle (\psi_+^\dagger \psi_-)(x,\tau)(\psi_-^\dagger \psi_+)(0,0)\rangle = A^4 \langle e^{2i\theta(x,\tau)} e^{-2i\theta(0,0)} \rangle_\theta$

$A = \frac{1}{\sqrt{2\pi a}}$　　a^{-1} - 截断　　$c = \frac{1}{\pi}$

$$\mathcal{L} = \frac{1}{2\pi}\left[(\partial_x\theta)^2 + (\partial_\tau\theta)^2\right] \qquad \pi_\theta = \partial_{\dot\theta}\mathcal{L} = \frac{\partial_\tau\theta}{\pi}$$

$$\Updownarrow$$

$$[\theta(x), \pi(x')] = -i\delta(x-x')$$

$$[\varphi(x), \theta(x')] = i\pi\Theta(x'-x) \qquad \pi_\theta = \frac{\partial_x\varphi}{\pi}$$

$$S = \int dx d\tau \left(\frac{(\partial_x\theta)^2}{2\pi} + \frac{(\partial_x\varphi)^2}{2\pi} + \pi i \partial_\tau\theta\partial_x\varphi \right)$$